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## SELF-CONSISTENT INCLUSION OF PION POLARISATION IN THE RELATIVISTIC DIRAC-BRUECKNER APPROACH TO NUCLEAR MATTER

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We investigate the influence of pion polarisation effects in the Dirac-Brueckner approach. The pion polarisation is included preserving the self-consistency of the DB approach. Results for single-particle properties, equation of state, and total effective cross sections in symmetric nuclear matter are presented. Also, we calculated the pion condensation threshold.

The properties of symmetric and asymmetric nuclear matter can be calculated in a relativistic many-body framework as offered by the Dirac-Brueckner approach [1,2]. The virtue of this particular approach is that it reproduces these properties, like saturation, compressibility and asymmetry energy in a very satisfactory way. The basic ingredients are a nucleon-nucleon interaction of the one-boson-exchange (OBE) type and a relativistic procedure to dress the nucleons in a nuclear medium as a consequence of effective two-nucleon interactions.

The extension we want to address in this letter is the influence of the nuclear medium on the exchanged mesons, in particular the pion. As is well known the pion propagator in a nuclear medium differs from the propagator in the vacuum. As a consequence the irreducible two-nucleon interaction itself is changed because of the exchange of a dressed pion. This may lead to a significant change of a number of fundamental quantities like the nucleon-nucleon collision rate in a nuclear medium [3,4]. For densities larger than a certain critical density the modification of the pion may be so strong that a pion condensate is formed [5-7]. For asymmetric nuclear matter even an undressed pion of mass  $m_\pi$  will give rise to pion condensation for certain asymmetries and densities when the difference of the neutron and proton chemical potentials exceed the pion mass  $\mu_n - \mu_p > m_\pi$  [8]. All these phenomena, connected to the dressing of the pion, have been studied for many years. However, within a particular approach it has

never been shown that pion condensation indeed occurs while all other properties of nuclear matter like saturation are still reproduced. We will present first results for symmetric nuclear matter within the Dirac-Brueckner (DB) framework. For more details on the DB method we refer to the existing literature (for a review see ref. [9] and references therein).

The effective pion propagator, which we denote as  $D(k)$  with  $k = (\omega, \mathbf{k})$  is the solution of a Dyson equation where  $\Pi(k)$  is the proper polarization insertion due to the nuclear medium, which in our case is a scalar

$$D(k) = [k^2 - m^2 - \Pi(k) + i\epsilon]^{-1}. \quad (1)$$

For a certain density, it may become possible that the pion propagator  $D(k)$  exhibits a pole at  $\omega=0$ , i.e., a particle-like excitation can exist with zero energy. When this occurs it becomes energetically favourable for the system to create particle-hole excitations with pion quantum numbers. This pion condensation and its consequences have been discussed by many authors [5-7].

We start by calculating the first-order contribution to  $\Pi$  (bubble diagrams) and include the higher-order effects due to the particle-hole interaction as outlined in ref. [10]. In the Dirac-Brueckner approach for nuclear matter only positive-energy solutions are included for the dressed nucleons which results in the absence of (infinite) vacuum polarisations. Also, no meson-retardation effects appear in the one-boson-exchange interaction due to the use of a particular

three-dimensional relativistic reduction of the Bethe–Salpeter equation for the nucleon–nucleon interaction. Therefore we only have to calculate  $\Pi(\omega, \mathbf{k})$  at  $\omega=0$ . The first-order polarisation has two contributions: the nucleon–nucleon particle–hole contribution  $\Pi_{\text{NN}}^{(1)}$  and the nucleon–delta ( $\Delta$ ) particle–hole contribution denoted as  $\Pi_{\text{N}\Delta}^{(1)}$ . The corresponding vertices  $\Gamma_{\text{NN}}$  and  $\Gamma_{\text{N}\Delta}$ , the values of the coupling constants, the form factors and the cut-off parameters were determined in ref. [2]. The values of the constants were fixed by a careful analysis of nucleon–nucleon elastic and inelastic data, pion production up to 1 GeV laboratory energy and including the correct  $\pi\text{N}$  resonant behaviour ( $\text{P}_{33}$  phase shift and inelasticity). For the  $\pi\text{NN}$  vertex we have a pseudo-vector coupling ( $g_\pi = 13.34(-)$ ,  $A_\pi^2 = 1.3 \text{ (GeV}^2\text{)}$ )

$$\Gamma_{\text{NN}} = g_\pi \left( \frac{A_\pi^2}{A_\pi^2 - k^2} \right) \gamma^5 \frac{\gamma^\mu k_\mu}{2m_N}, \quad (2)$$

and the  $\pi\text{N}\Delta$  vertex is given as ( $f_{\Delta\pi} = 2.09(-)$ ,  $A_\Delta^2 = 0.83 \text{ (GeV}^2\text{)}$ )

$$\Gamma_\mu = \frac{f_{\Delta\pi}}{m_\pi} \left( \frac{A_\Delta^2}{A_\Delta^2 - k^2} \right) k_\mu. \quad (3)$$

For the Lindhard function corresponding to the  $\Pi_{\text{NN}}^{(1)}$  contribution we find

$$\Pi_{\text{NN}}^{(1)}(k) = i \int \frac{d^4p}{(2\pi)^4} \text{Tr}[G_N(p) \Gamma_{\text{NN}} G_N(p+k) \Gamma_{\text{NN}}], \quad (4)$$

with  $G_N(p)$  the full dressed nucleon propagator

$$G_N(p) = \frac{(\gamma^\mu p_\mu + m_N^*)}{E_p^*} \times \left( \frac{1}{2} \frac{1}{p_0^* - E_p^* + i\epsilon} + i\pi \delta(p_0^* - E_p^*) \theta(k_F - |\mathbf{p}|) \right), \quad (5)$$

where  $k_F$  is the Fermi momentum (the trace is also taken over the suppressed isospin indices). The quantities labeled with an asterisk (\*) denote dressed quantities, i.e. modified by the medium interactions as prescribed in the self-consistent Dirac–Brueckner prescription. Here one calculates the nucleon self-energy  $\Sigma(k) = \Sigma_s(k) + \gamma^0 \Sigma_0(k)$  with scalar ( $\Sigma_s$ ) and vector ( $\Sigma_0$ ) components. We have neglected the small

cartesian vector contribution  $\Sigma_v(k)$ . It is understood that a *self-consistent* Dirac–Brueckner calculation of the fields  $\Sigma_s$  and  $\Sigma_0$  has been performed *including* the effects of the pion polarisation. Thus the resulting fields  $\Sigma_s$  and  $\Sigma_0$  are different from the ones used before [2] since now the dressed pion propagator based on the full  $\Pi$  has been taken into account.

The dressed quantities appearing in (5) are the effective mass  $m_N^*$ , the effective energy  $E_p^*$  and momentum  $p_\mu^*$ . They are given as

$$m_N^* = m_N + \Sigma_s, \quad E_p^* = (p^2 + m_N^{*2})^{1/2}, \\ p_\mu^* = p_\mu + \delta_{0\mu} \Sigma_0. \quad (6)$$

The expression (5) for  $\Pi_{\text{NN}}^{(1)}$  can be worked out to give (we recall that for the present purpose we have set  $\omega=0$ ):

$$\text{Re } \Pi^{(1)}(\mathbf{k}) = - \frac{g_\pi^2}{m_N^2} \left( \frac{A_\pi^2}{A_\pi^2 + \mathbf{k}^2} \right)^2 \int \frac{d^3p}{(2\pi)^3} \theta(k_F - |\mathbf{p}|) \\ \times \left( \frac{1}{E_p^*} + \frac{1}{E_{p+k}^*} \right) \left( \mathbf{p} \cdot \mathbf{k} + \frac{2(m_N^*)^2 \mathbf{k}^2}{k^2 + 2\mathbf{p} \cdot \mathbf{k}} \right). \quad (7)$$

The Lindhard function corresponding to the  $\Pi_{\text{N}\Delta}^{(1)}$ -contribution is given by:

$$\Pi_{\text{N}\Delta}^{(1)}(k) = i \int \frac{d^4p}{(2\pi)^4} \text{Tr}[G_\Delta^{\mu\nu}(p) \Gamma_\nu G_N(p+k) \Gamma_\mu], \quad (8)$$

with  $G_N$  again the full dressed nucleon propagator (5) and for the delta propagator  $G_\Delta^{\mu\nu}$  in the Rarita–Schwinger representation [2] we have taken the positive-energy part only:

$$G_\Delta^{\mu\nu} = \left( -g^{\mu\nu} + \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{p^\mu \gamma^\nu - p^\nu \gamma^\mu}{3m_\Delta} + \frac{2(p^\mu p^\nu)}{3m_\Delta^2} \right) \\ \times \frac{1}{2E_p^\Delta} \frac{1}{p_0 - E_p^\Delta + i\epsilon}. \quad (9)$$

The expression (8) can be reduced to the form

$$\text{Re } \Pi^{(1)}(\mathbf{k}) = - \frac{4}{3} \frac{f_{\Delta\pi}^2}{m_\pi^2} \left( \frac{A_\Delta^2}{A_\Delta^2 + \mathbf{k}^2} \right)^4 \\ \times \int \frac{d^3p}{(2\pi)^3} \frac{\text{Tr}[p, p+k]}{E_{p+k}^\Delta E_p^\Delta} \frac{\theta(k_F - |\mathbf{p}|)}{E_p^* - \Sigma_0 - \Sigma_{p+k}^\Delta}, \quad (10)$$

$$\text{Tr}[p, p+k] = \frac{4}{3} \left( -k^2 + \frac{[\mathbf{k} \cdot (\mathbf{p} + \mathbf{k})]^2}{m_\Delta^2} \right) \times (m^{*2} - \Sigma_0 E_p^* + m_\Delta m^* - \mathbf{p} \cdot \mathbf{k}). \quad (10 \text{ cont'd})$$

Note that we have treated the  $\Delta$  as an elementary stable particle ( $m_\Delta = 1.23 \text{ GeV}/c^2$ ,  $\Gamma_\Delta = 0$ ). At present this is a minor approximation as we have checked explicitly. Both  $\Pi_{NN}^{(1)}$  and  $\Pi_{N\Delta}^{(1)}$  have zero imaginary parts and reduce to the non-relativistic expressions as used in ref. [10] except for the form factors. The choice of the form factors at the interaction vertices influences the values of the polarisation considerably. In particular the dipole form factor on the  $\pi N\Delta$  vertex reduces  $\Pi_{N\Delta}$  as compared to the conventional monopole form factor for instance used by e.g. Dickhoff et al. [10]. The justification of the dipole form factor in the Dirac-Brueckner approach is that it is needed to render the high momentum contributions finite [2]. As we already mentioned, all free parameters and thus implicitly the choice of the form factor were fitted to NN data up to 1 GeV. Also the high-momentum behaviour at the vertices is different in the non-relativistic case as compared to the relativistic one. Both these considerations are taken into account in the choice of the form factor.

To include higher-order effects in  $\Pi$  one has to incorporate the particle-hole interactions, which we denote as  $H(k)$ . The full polarisation can then be written as

$$\Pi(k) = \Pi^{(1)}(k) + \Pi^{(1)}(k)H(k)\Pi(k). \quad (11)$$

Commonly the particle-hole interaction  $H(k)$  is written as  $g'_{ij}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)/k^2$ . The indices  $ij$  indicate that the particle-hole interaction couples, in principle, differently to  $\Pi_{NN}^{(1)}$  and  $\Pi_{N\Delta}^{(1)}$ . The evaluation of  $g'_{ij}$  is an intricate business and yet not settled. The most elaborated approach (although in a non-relativistic model) is found in ref. [11]. However, no results for the equation of state are given, so we do not know if the calculation produces saturation at all. The effective mass shows a peculiar density dependence which we do not find. It is obtained by means of the Landau definition. This can only be justified if the model for the calculation fulfills certain criteria, i.e. there exists a specific functional relationship between the total energy, single-particle energy and effective two-particle interaction. The use of the

Brueckner  $G$ -matrix as the residual interaction in the treatment of pion polarisation does not obey these criteria. In a relativistic approach ref. [12] finds values of  $g'$  that are similar to non-relativistic ones. In this calculation only the first part of the self-consistent Dirac-Brueckner equations is used. Their OBE interaction (HEA potential) contains pseudo-scalar pion-exchange whereas we use pseudo-vector. A full self-consistent insertion of pion polarisations should also take self-consistency effects on  $g'_{ij}$  into account. Clearly this is a tremendous task. In order to simplify the model we take  $g'_{ij}$  equal and constant. This does no right to a possible density dependence of, or difference (although one can always define an averaged  $g'_{av}$ , as is done in ref. [10]) in  $g'_{ij}$ . With this we can write the proper self-energy  $\Pi(k)$  in (11) as:

$$\Pi(k) = (\Pi_{NN}^{(1)} + \Pi_{N\Delta}^{(1)}) / [1 - (g'/k^2)(\Pi_{NN}^{(1)} + \Pi_{N\Delta}^{(1)})]. \quad (12)$$

In the literature [6,10,12-14] one finds theoretical values for  $g'_{ij}$  ranging from 0.4 to 0.6, the experimental value is found to be between 0.6 and 0.7. Motivated by the indications for additional screening upon inclusion of the induced interaction [11] we choose  $g'_{av}$  conservatively and set it equal to 0.6. Since the dependence of the calculated quantities on  $g'$  turns out to be rather weak in the range  $0.5 < g' < 0.7$ , we think this simplification does not affect the general trends we observe. (For  $g' = 0.5$ , respectively  $g' = 0.7$ , we find a shift compared to the values of  $E_b$  in table 1 of +0.3, -0.3 MeV at  $k_F = 0.21$  and +3, -1 at  $k_F = 0.35$ .) The insertion of the result (12) into the expression (1) for the dressed pion propagator finalises our treatment of medium effects on the pion. The effective, in-medium, interactions between nucleons as described by the Dirac-Brueckner  $G$ -matrix now contains, self-consistently, the exchange of dressed pions.

However, simply replacing all pion propagators by effective pion propagators as is done in ref. [15], is incorrect. In the evaluation of the nucleon self-energy we should be careful to avoid double countings. These originate from the fact that we obtain the self-energy (a two-point function) by closing the four-point  $G$ -matrix with a hole line. A closer examination reveals that the problem arises from the second-order "direct" (Hartree) diagram (fig. 1a) and the first-order "exchange" (Fock) diagram (fig. 1d). Figs. 1b and

Table 1

Single-particle values at the Fermi surface and the binding energy, including pion polarizations ( $g' = 0.6$ ), versus nuclear matter density as expressed by the Fermi momentum  $k_F$ .

$k_F$ (GeV/c)	$m_{DB}^*$	$m_{DB+POL}^*$ (GeV/c <sup>2</sup> )	$\Sigma_{DB+POL}^S$ (GeV)	$\Sigma_{DB+POL}^0$ (GeV)	$\Sigma_{DB+POL}^V$ (GeV)	$E_{b, DB+POL}$ (MeV)
0.20	0.744	0.733	-0.219	-0.169	-0.020	-11.2
0.22	0.695	0.679	-0.272	-0.214	-0.017	-12.3
0.24	0.643	0.621	-0.333	-0.266	-0.022	-13.4
0.26	0.588	0.565	-0.394	-0.319	-0.033	-14.4
0.28	0.530	0.510	-0.454	-0.372	-0.046	-15.0
0.34	0.376	0.365	-0.616	-0.549	-0.103	-4.7
0.36	0.329	0.312	-0.664	-0.634	-0.117	7.1

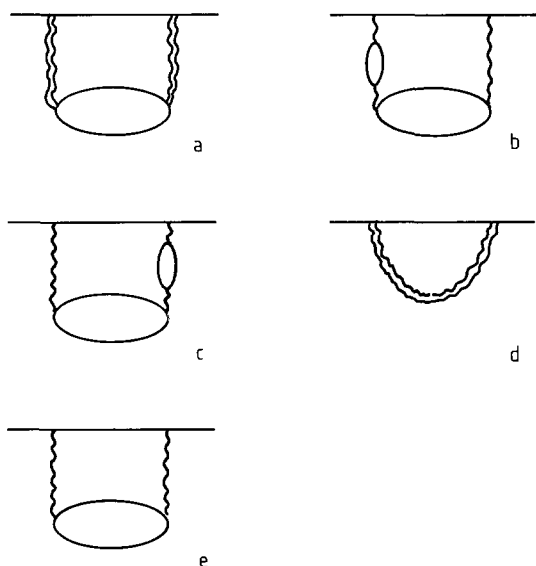


Fig. 1. Various contributions to the nucleon self-energy when this is calculated with the Brueckner  $G$ -matrix including pion polarizations. A wiggly line stands for a bare pion propagator, a double wiggly line for a full pion propagator.

1c show first-order self-energy insertions in, respectively, the left and right pion propagator. They are topological the same, but are both included in fig. 1a. Also fig. 1e, stemming from fig. 1d by putting in the first-order self-energy in the pion propagator, is included in fig. 1a. We solved this double-counting problem by subtracting from the Dirac-Brueckner  $G$ -matrix the appropriate diagrams. This defined a modified  $G$ -matrix, denoted  $G'$ , from which the nucleon self-energy  $\Sigma$  can be calculated without double countings. In any case, if one wants to calculate four-point observables the full  $G$ -matrix instead of  $G'$  has

to be used. This implies that certain (important) diagrams do not contribute in the same way whether used in four-point observables (like effective nucleon-nucleon collision rates) as compared to two-point observables (like the self-energy or nucleon single-particle energy).

The pion dispersion relation  $\omega^2(k) = m_\pi^2 + k^2 + \text{Re } \Pi(k)$  exhibits the usual behaviour as a function of density. At sufficient high densities and at low momenta  $k$  we have the free pion spectrum and for momenta equal 2–3  $m_\pi$  the pionic branch is dominant [4,13]. We also calculated the pion condensation threshold as a function of the particle-hole interaction strength  $g'$ . The result is displayed in fig. 2, for comparison a typical non-relativistic result [16] is also represented. In comparing our result with that of ref. [16] one may argue that the values of the effective masses are entirely different. However, it should be realised that the  $m^*$  values quoted in tables 1 and 2 are the relativistic effective masses which are defined differently. To obtain the corresponding “non-relativistic” values one should consider  $(m^{*2} + k_F^2)^{1/2}$ , which is then not so different. For reasonable values of  $g'$  we find pion condensation not to appear below three times normal nuclear matter density. The difference with the non-relativistic result is mainly due to two effects that reduce the polarisation: (a) the dipole form factor on the  $\pi N \Delta$  vertex and (b) the self-consistent effective mass, which decreases with density.

We calculated the equation of state (EOS) for  $g' = 0.6$ , the result is shown in fig. 3 and the single-particle values are given in table 1. We observe that, while the self-energy and effective mass differ in the order of a few tens of MeV from the earlier DBHF

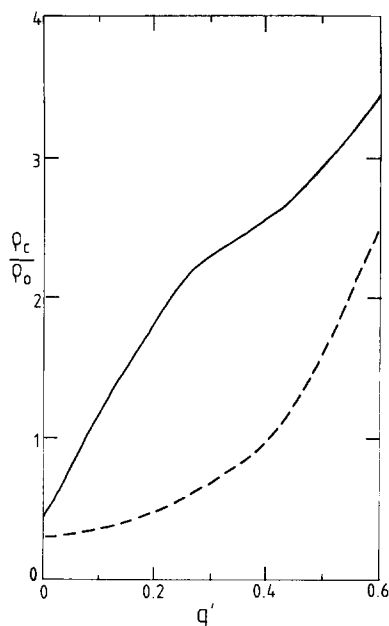


Fig. 2. The pion-condensation threshold as a function of the particle-hole interaction strength  $g'$ . The dashed line is the non-relativistic result obtained in ref. [16]. The full line is our result.

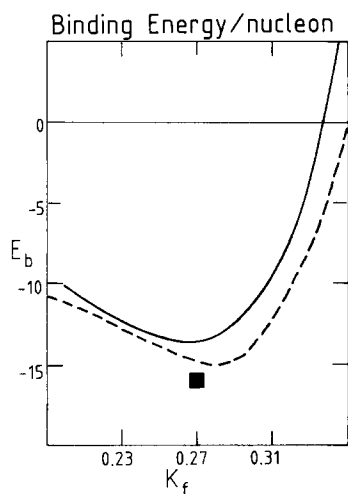


Fig. 3. Binding energy per nucleon versus Fermi momentum. The full line is the DBHF result of ref. [2] not including pion polarisation. The dashed line is the result with pion polarizations and  $g' = 0.6$ . The empirical saturation point is also indicated.

results without pion polarisation [2], the shift in binding energy is small: about 1 MeV at saturation, towards the empirical saturation point. The EOS is slightly softened, the compression modulus is around

220 MeV. Also we do not observe a dramatic behaviour of the EOS as we approach the pion condensation threshold, although we were not able to calculate too close to the threshold. The dependence of the binding energy on the particle-hole interaction strength  $g'$  is rather unexpected. As we decrease  $g'$ , the pion interaction is enhanced and one would expect more binding, since the pion is an attractive meson. It turns out that decreasing  $g'$  causes a decrease of the binding energy. The effective mass, however, decreases so it may very well be possible that this effect is due to a further "quenching" of the (attractive) scalar part of the interaction mechanism as indicated in ref. [2]. Also the attractive term in the second-order pion exchange diagram is reduced by a smaller effective mass.

To demonstrate the behaviour of four-point quantities upon insertion of the pion polarisation we also calculated nn and np total effective cross sections in the same way as discussed in ref. [17]. Note that we did not include  $\Delta$ -particle intermediate states in the  $G$ -matrix, so the model does not reproduce accurately NN-phase shifts for  $P_{\text{LAB}} > 0.85$ . For the NN cross sections however, this leads only to minor differences. The results are displayed in table 2. We see that these are greatly increased, by a factor  $\sim 2(\sigma_{\text{PP}})$  to  $\sim 4(\sigma_{\text{NP}})$  compared to the DBHF results without pion polarisation [17]. The latter were found to be smaller (by a factor 2–3 depending on the density and momentum) than the free ones [17]. Therefore we find a net increase of a factor 1–2 as compared to the free cross sections.

We think an explanation of the drastic change in the cross sections as compared to the moderate shifts in the self-energy can be found in the following. First we remind that the self-energy is a function of  $G$ , whereas the cross sections go by  $|G|^2$ . The total effective cross sections calculated in the Born approximation with the one boson exchange interaction show already an increase by a factor  $\sim 1.5(\sigma_{\text{NN}})$  and  $\sim 2.5(\sigma_{\text{NP}})$ , compared to the result with no pion polarisations included. The  $G$ -matrix elements dominated by one-pion exchange increase by a factor 1–3. However, the contribution of these matrix elements to the nucleon self-energy is in the order of a few tens of MeV. But this is to be compared with the total self-energy, typically in the order of a few hundred MeV. Also we have calculated values for the "mean field"

Table 2

Values of the single-particle interaction and effective isospin averaged total cross sections, including pion polarizations ( $g' = 0.6$ ). For comparison the DB values and free cross sections of ref. [17] are also given.

$\rho$	$k_f$ (GeV/c)	$m^*$ (GeV/c <sup>2</sup> )	$P_{\text{LAB}}$ (GeV/c)	$\Sigma_s$ (GeV)		$\Sigma_0$ (GeV)		Re $U_{\text{DB+POL}}$ (MeV)	Re $U_{\text{DB}}$ (MeV)	$\sigma_{\text{DB+POL}}$ (mb)	$\sigma_{\text{DB}}$ (mb)	$\sigma_{\text{FREE}}$ (mb)
				Re	Im	Re	Im					
$\frac{1}{2}\rho_0$	0.21	0.708	0.38	-0.274	0.012	-0.236	0.022	-15	-33	162	61.0	83.3
			0.46	-0.249	-0.026	-0.208	-0.019	-12	-26	108	35.0	56.8
			0.65	-0.175	-0.007	-0.131	-0.003	-9	-15	68.7	20.9	33.3
			0.85	-0.151	0.023	-0.105	0.019	-4	-7	48.9	18.7	24.7
			1.25	-0.153	0.045	-0.094	0.032	6	4	32.5	18.5	18.5
			1.65	-0.152	0.050	-0.087	0.032	16	14	26.3	20.1	18.1
$\rho_0$	0.27	0.537	0.38	-0.404	-0.002	-0.329	0.009	-33	-48	147	34.7	88.5
			0.46	-0.391	0.000	-0.321	0.006	-17	-38	95.4	24.4	58.8
			0.65	-0.316	-0.007	-0.240	-0.007	-6	-18	55.3	17.7	33.6
			0.85	-0.270	0.030	-0.191	0.024	5	-2	44.3	17.1	24.8
			1.25	-0.254	0.078	-0.164	0.054	23	16	33.2	17.6	18.6
			1.65	-0.257	0.097	-0.156	0.062	40	35	32.1	20.3	18.1
$2\rho_0$	0.335	0.378	0.38	-0.592	0.004	-0.515	0.006	-5	-4	70.3	22.1	99.1
			0.46	-0.561	0.009	-0.482	0.012	8	1	60.7	20.8	63.2
			0.65	-0.497	0.028	-0.407	0.031	33	22	43.5	20.0	34.0
			0.85	-0.443	0.055	-0.340	0.054	51	38	36.3	21.6	24.9
			1.25	-0.401	0.108	-0.290	0.094	82	72	32.1	24.0	18.9
			1.65	-0.393	0.137	-0.276	0.110	106	101	30.5	25.8	18.5
$2.5\rho_0$	0.365	0.301	0.38	-0.677	0.000	-0.653	0.000	65	66	69.1	25.9	103
			0.46	-0.637	0.003	-0.597	0.010	68	63	61.9	24.1	65.8
			0.65	-0.560	0.030	-0.501	0.050	87	80	48.7	24.9	34.3
			0.85	-0.502	0.057	-0.423	0.072	102	101	39.2	24.5	24.9
			1.25	-0.447	0.109	-0.354	0.117	134	141	33.8	25.9	18.9

which we denote as Re  $U$ , and which is obtained as the difference of the single-particle energy and the free energy:

$$\text{Re } U = [k^2(1 + \Sigma^v)^2 + (m_N + \Sigma^s)^2]^{1/2} - \Sigma^0 - (k^2 + m_N^2)^{1/2}, \quad (13)$$

where only the real parts of  $\Sigma(k)$  are included. From the values quoted in table 2 one finds small differences with the DB results [17].

In conclusion, we presented a self-consistent inclusion of pion self-energy contributions (pion polarizations) in the DBHF model, avoiding double countings. This resulted in a small shift ( $\sim 1$  MeV at saturation) of the EOS towards the empirical saturation point. Also we calculated effective cross sections, these increased up to a factor five compared to the earlier DBHF results. Compared to the free cross sections the enhancement is of the order of 1–2. This

might introduce interesting effects in nucleus–nucleus collisions such as the sideward flow in relativistic heavy ion collisions [18].

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